

Business PreCalculus MATH 1643 Section 004, Spring 2014
Lesson 13: Functions

Definition 1. Relation: Any set of ordered pairs is called a relation. The set of all first components is called the domain of the relation, and the set of all second component is called the range of the relation.

Example 1. Find the domain and range of the relation
 $\{(Titanic, 600.8), (Star Wars IV, 461), (Shrek 2, 441.2),$
 $(E.T., 435.1), (Star Wars I, 431.1), (SpiderMan, 403.7)\}$

Solution:

The domain is the set of all first components, or
 $\{Titanic, StarWarsIV, Shrek2, E.T., StarWarsI, SpiderMan\}$.
The range is the set of all second components, or
 $\{600.8, 461, 441.2, 435.1, 431.1, 403.7\}$.

Definition 2. Function: A function is a relation in which each element of the domain corresponds to one and only one element of the range. So the relation in example 1 defines a function.

Example 2. The relation $\{(Titanic, 600.8), (Star Wars IV, 461), (Shrek 2, 441.2),$
 $(E.T., 435.1), (Star Wars IV, 431.1)\}$ is **not a function** because the element (Star Wars IV) in the domain corresponds to two elements from the range, namely, 461 and 431.1.

Definition 3. Functions Defined by Equations: Functions can be defined using equations. For example, the equation

$$y - x^2 = 0$$
$$y = x^2$$

defines a function. We can replace the dependent variable, in this case y , with functional notation $f(x)$ and express the function as

$$f(x) = x^2 \quad (\text{read } f \text{ of } x \text{ equals } x^2).$$

Note that in this notation x is an element of the domain and $f(x)$ is an element of the range.

Definition 4. Evaluating a Function: Let f be the function defined by the equation

$$y = x^2 - 6x + 8$$

Evaluate each function value: $f(3)$, $f(-2)$, $f(x + h)$.

Solution: Because the function is called f , we replace y by $f(x)$ and write

$$f(x) = x^2 - 6x + 8$$

In this notation, the independent variable x is a placeholder. We can write $f(x) = x^2 - 6x + 8$ as $f() = ()^2 - 6() + 8$. So

$$f(3) = (3)^2 - 6(3) + 8$$

$$f(3) = 9 - 18 + 8 = -1.$$

$$f(-2) = (-2)^2 - 6(-2) + 8$$

$$f(-2) = 4 + 12 + 8 = 24.$$

$$f(x + h) = (x + h)^2 - 6(x + h) + 8$$

$$f(x + h) = x^2 + 2xh + h^2 - 6x - 6h + 8.$$

Definition 5. Vertical Line Test: *If no vertical line intersects the graph of a curve at more than one point, then the curve is the graph of a function.*